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Math 238 Test # 2

No aids: Closed book. You are not permitted to consult with your fellow students in any way. Time: 80 minutes.

Question 1. (4 pts) Solve the initial value problem

$$y' = \frac{-}{y}, \quad y(0) = -.$$

Question 2. (3 pts) Find a continuous solution to the initial value problem on $[1, 3]$:

$$y' + \frac{1}{t}y = \begin{cases} 3t, & 1 \le t \le 2 \\ 0, & 2 < t \le 3 \end{cases} \qquad y(1) = 1$$

Question 3. (2 pts) The initial value problem $y' = \sqrt{|y|}$, $y(0) = 0$ has two solutions

$$y \equiv 0 \text{ and } y^* = \begin{cases} t^2/4, & t \ge 0 \\ -t^2/4, & t < 0 \end{cases}$$

Why doesn't the existence and uniqueness theorem of Chapter 3 work in this case?

Question 4. (2 pts) Solve the Bernoulli differential equation $y' + \frac{y}{t} = t^2 y^2$ by finding an appropriate substitution $v = \frac{1}{y}$ which linearizes the problem.

Question 5. (4 pts) Solve the exact differential equation

$$1 + y^2 + 2(t+1)yy' = 0, \quad y(0) = 1.$$

Question 6. (4 pts) A particle of mass m falls into a stream of air moving with constant velocity v_0 and is carried along with it. The particle is acted on by a force $k(v - v_0)^2$. Determine the distance x travelled and the velocity v as a function of time t under the initial conditions $x(0) = v(0) = 0$.

Question 7. (3 pts) Show that $\{y_1 = t^2, y_2 = t^3\}$ is a fundamental set of $t^2y'' - 4ty' + 6y = 0$ for all $t > 0$ and all $t < 0$. Show that $\{y_1 = t^2, y_2 = t^3\}$ is in fact a fundamental set for all $-\infty < t < \infty$, were we understand that any legitimate solution has to be smooth (all derivatives are continous functions for any t).

Compute the Wronskian $W[y_1, y_2](t)$. What is the value of $W[y_1, y_2](0)$? Is this a contradiction?